Mécanique des milieux continus

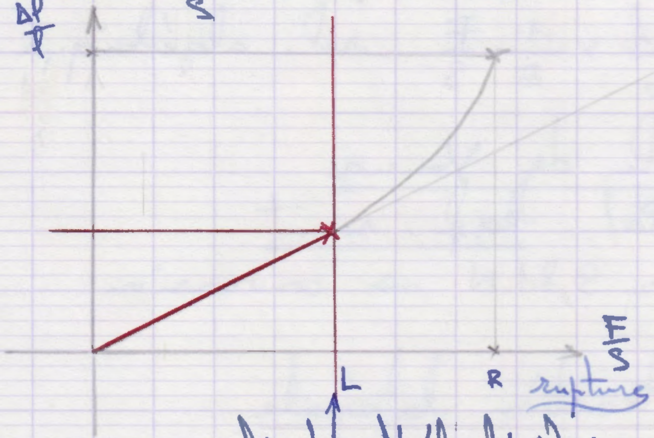
A- Notion d'élasticité

déformation ⟷ forces

effet   cause

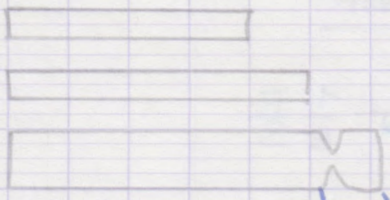
$\frac{\Delta l}{l}$ allongement relatif

contrainte $\frac{F}{S}$ ($N.m^{-2}$ ou Pa)



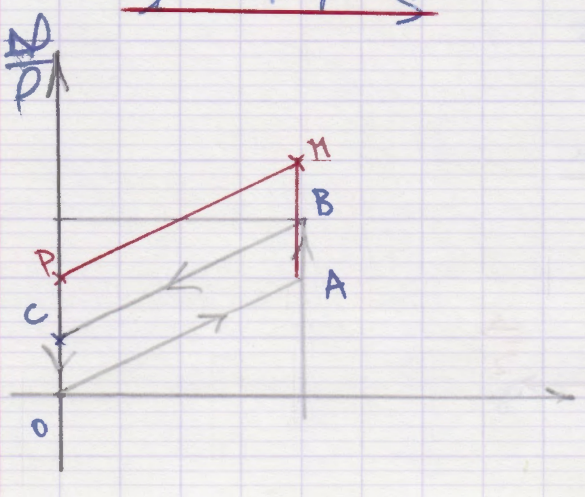
limite d'élasticité

ductilité



striction

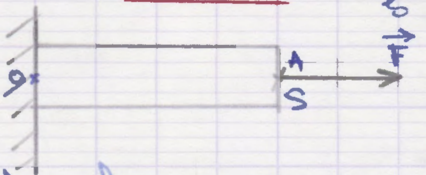
$\frac{\Delta l}{l}$ fct $\frac{F}{S}$ loi de Hooke



B_ Traction et compression

1- Traction

$\frac{\Delta l}{l_0} = \frac{1}{E}\frac{F}{S}$



l_0 longueur
a_0 dim transversale } initialement

$\frac{\Delta l}{l_0} = \frac{l-l_0}{l_0} = \frac{1}{E}\frac{F}{S}$ rel. algébrique

E module de Young

E ($\frac{N}{m^2}$ ou Pa) $kgf \cdot mm^{-2}$

1 kgf = 9,81 N → conversion

2- Contraction latérale

$l > l_0$ $\frac{a_0-a}{a_0} = \sigma \frac{l-l_0}{l_0}$

$a < a_0$

σ coef. de POISSON

$\Delta a = a - a_0$

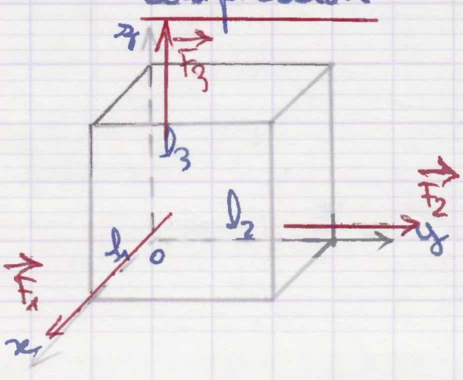
$\frac{\Delta a}{a} = -\sigma \frac{\Delta l}{l} = -\sigma \frac{1}{E}\frac{F}{S}$

$\frac{\Delta a}{a} = -\frac{\sigma}{E}\frac{F}{S}$

$\frac{\Delta V}{V} = \frac{\Delta S}{S} + \frac{\Delta l}{l_0} = 2\frac{\Delta a}{a_0} + \frac{\Delta l}{l_0} = -\frac{2\sigma}{E}\frac{F}{S} + \frac{1}{E}\frac{F}{S} = \frac{(1-2\sigma)}{E}\frac{F}{S} > 0$ ↑ exp^{ce}

donc $\sigma < 0,5$

3- Compression



$$N_1 = \frac{F_1}{l_2 l_3} \qquad N_2 = \frac{F_2}{l_3 l_1} \qquad N_3 = \frac{F_3}{l_1 l_2}$$

$F_1 \to N_1$

$F_2 = F_3 = 0$

$$a_{11} = \frac{1}{E} N_1 \qquad a_{21} = -\frac{\sigma}{E} N_1 \qquad a_{31} = -\frac{\sigma}{E} N_1$$

face ↗ cause ↖

$F_2 \to N_2$

$F_1 = F_3 = 0$

$$a_{12} = -\frac{\sigma}{E} N_2 \qquad a_{22} = \frac{1}{E} N_2 \qquad a_{32} = -\frac{\sigma}{E} N_2$$

$F_3 \to N_3$

$F_1 = F_2 = 0$

$$a_{13} = -\frac{\sigma}{E} N_3 \qquad a_{23} = -\frac{\sigma}{E} N_3 \qquad a_{33} = \frac{1}{E} N_3$$

face 1
$$a_1 = a_{11} + a_{12} + a_{13} = \frac{N_1}{E} - \frac{\sigma}{E} N_2 - \frac{\sigma}{E} N_3$$

$$a_2 = a_{21} + a_{22} + a_{23} = -\frac{\sigma}{E} N_1 + \frac{N_2}{E} - \frac{\sigma}{E} N_3$$

$$a_3 = a_{31} + a_{32} + a_{33} = -\frac{\sigma}{E} N_1 - \frac{\sigma}{E} N_2 + \frac{N_3}{E}$$

$$\theta = \frac{\Delta V}{V} = a_1 + a_2 + a_3 = \frac{1}{E}(N_1 + N_2 + N_3) - \frac{2\sigma}{E}(N_1 + N_2 + N_3)$$

$$= \left(\frac{1-2\sigma}{E}\right)(N_1 + N_2 + N_3)$$

compression : $N_1 = N_2 = N_3 = -p$

$$\theta = -3\left(\frac{1-2\sigma}{E}\right) \cdot p$$

coef. de compressibilité : $\boxed{\chi = -\frac{\theta}{p} = 3\left(\frac{1-2\sigma}{E}\right)}$

matériau E, σ, χ

$E, \sigma \rightarrow \lambda, \mu$

Coefficients de Lamé

$$a_1 = \frac{1}{E}(N_1 + \sigma N_1 - \sigma N_1 - \sigma N_2 - \sigma N_3)$$

$$a_1 = \frac{1}{E}\left[(1+\sigma)N_1 - \sigma\underbrace{(N_1 + N_2 + N_3)}_{\frac{\Theta E}{1-2\sigma}}\right]$$

$$a_1 E = (1+\sigma)N_1 - \frac{\sigma E \Theta}{1-2\sigma}$$

$$N_1 = a_1 \cdot \frac{E}{1+\sigma} + \frac{\sigma E \Theta}{(1-2\sigma)(1+\sigma)}$$

$$\boxed{N_1 = \lambda a_1 + 2\mu\Theta}$$

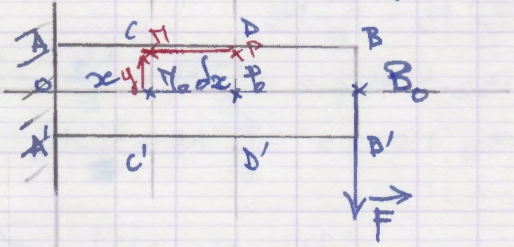
$$N_2 = \lambda a_2 + 2\mu\Theta$$

$$N_3 = \text{———}$$

$\lambda =$ ——— $\rightarrow E = (\lambda, \mu)$

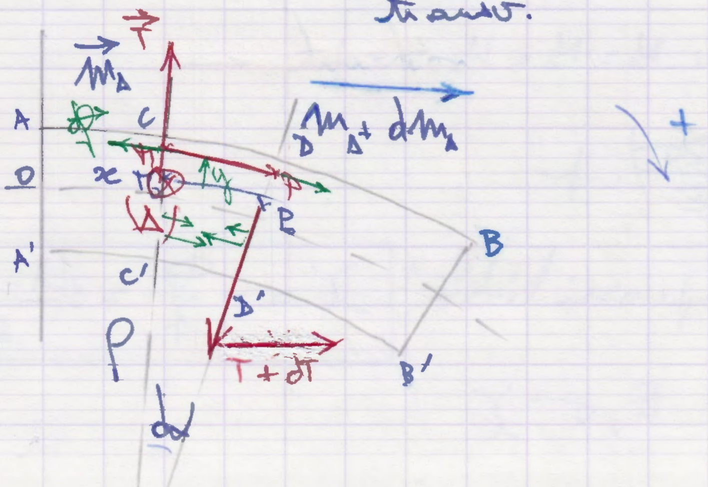
$\mu =$ ——— $\quad \sigma = (\lambda, \mu)$

4. Flexion plane



× $\vec{F} \rightarrow$ longitudinal $\rightarrow$ traction
$\rightarrow$ transversale

× $\vec{M} \rightarrow$ long. $\rightarrow$ torsion
$\rightarrow$ transv.



$dM_\Delta = df \cdot y$

OB_0 = ligne neutre

$M_0P_0 = dx = \rho\, d\alpha$

$MP = (\rho + y)\, d\alpha$

$\frac{MP - M_0P_0}{M_0P_0}$ al. relatif $= \frac{1}{E} \frac{df}{ds} = \frac{y\, d\alpha}{\rho\, d\alpha} = \frac{y}{\rho}$

$\Rightarrow df = \frac{y}{\rho} E \cdot ds$

$dM_\Delta = df \cdot y = \frac{E}{\rho} y^2\, ds$

$M_\Delta = \frac{E}{\rho} \underbrace{\int y^2\, ds}_{\text{moment quadratique } I} = \frac{E \cdot I}{\rho} = \boxed{E \cdot I \frac{d\alpha}{dx} = M_\Delta}$ moment fléchissant

T effort tranchant

$-\cancel{M_\Delta} + \cancel{M_\Delta} + dM_\Delta + (T + dT)\, dx = 0$

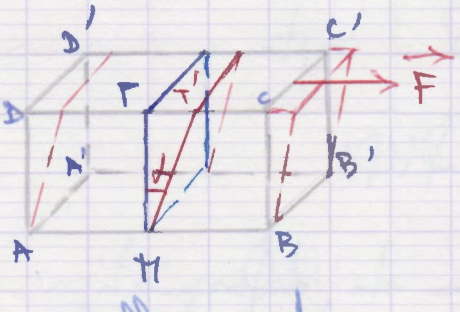
$dM_\Delta + T dx = 0$

$$\boxed{T = -\frac{dM_\Delta}{dx}}$$

dérivée du moment fléchissant par rapport à la distance dx

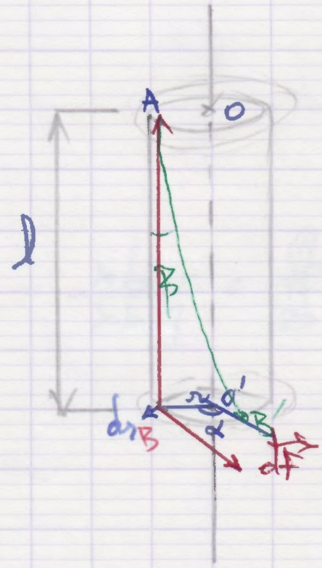
5. <u>Torsion</u>

cisaillement



module de cisaillement ou de rigidité

$$\boxed{\alpha = \frac{1}{G} \frac{dF}{dS}}$$



$$\beta = \frac{\overline{BB'}}{AB} = \frac{r.\alpha}{l} = \frac{1}{G}\frac{dF}{dS} \rightarrow dF = \frac{r\alpha G}{l}\,2\pi r\,dr$$

$$dS = 2\pi r\,dr \qquad dF = \frac{2\pi G}{l}\,\alpha.\,r^2\,dr$$

$$dM_{oo'} = dF \times r = \frac{2\pi G}{l}\,\alpha.\,r^3\,dr$$

$$M_{oo'} = \int dM = \frac{2\pi G}{l}\,\alpha \int_0^R r^3\,dr = \frac{2\pi G}{l}\,\frac{R^4}{4}.\,\alpha = C.\alpha$$

$\boxed{C = \frac{\pi G R^4}{2l}}$ constante de torsion

gdeur φ → capteur → très souvent V

$V = f(\varphi)$

Capteurs de déformation

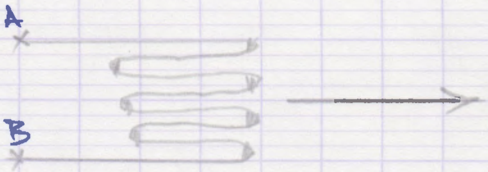
jauges de contraintes

Extensométrie $\quad R = \boxed{\rho}\,\boxed{\frac{l}{S}}$

$$\begin{matrix} F \\ S \end{matrix} \xrightarrow{\text{structure}} \frac{\Delta l}{l} \longrightarrow \left(\frac{\Delta R}{R}\right) \longrightarrow (V) \text{ tension}$$

corps d'épreuve — capteur (jauge de contrainte) — conditionneur

$$R = n.\rho \frac{l}{S}$$



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{ds}{s}$$

$$\frac{d\rho}{\rho} = C \frac{dV}{V}$$ (C cte Brigmann) x

$$\frac{ds}{s} = 2\frac{da}{a} = -2\sigma \frac{dl}{l}$$

$$\frac{d\rho}{\rho} = C.\left(\frac{dS}{S} + \frac{dl}{l}\right) = -2\sigma C \frac{dl}{l} + C\frac{dl}{l}$$

$$\frac{dR}{R} = \underbrace{C(1-2\sigma)\frac{dl}{l}}_{\frac{dl}{l}} + \frac{dl}{l} + 2\sigma\frac{dl}{l}$$

$$\frac{dR}{R} = \left[C(1-2\sigma) + (1+2\sigma)\right]\frac{dl}{l}$$

$$\frac{dR}{R} = K.\frac{dl}{l}$$

K facteur de jauge